Matrix Population Models for Wildlife Conservation and Management

27 February - 5 March 2016

Jean-Dominique LEBRETON Jim NICHOLS
Madan OLI Jim HINES

UF UNIVERSITY of FLORIDA

Lecture 9
Demographic stochasticity


Krishna ATHREYA


Peter NEY


Peter JAGERS


Eugene SENETA

Major contributors to the theory of branching processes,
a natural framework for modeling demographic stochasticity

A simple example

- Death / Survival as a coin tossing experiment
- Reproduction as a discrete distribution
  (e.g. Clutch size ≈ Poisson distribution)
- Independence of individuals

- As usual, "random" stands for residual variation after adequate stratification
- Independence may be more restrictive

Death/Survival

In what follows, everything is conditional on $N_1(t)$ and $N_2(t)$

Year t Year t+1

$N_1 \xrightarrow{s_1} N_2$

$N_2 \xrightarrow{s_2} N_2$

$N_2(t+1) \approx Bin(N_1(t),s_1) + Bin(N_2(t),s_2)$

Reproduction

Year t Newborn $N_0(t)$ Year t+1

$N_1 \xrightarrow{f_1} N_0$

$N_2 \xrightarrow{f_2} N_0$

$N_0 \xrightarrow{s_0} N_1$

$N_0(t) \approx Poisson(N_1(t)f_1) + Poisson(N_2(t)f_2)$
$N_1(t+1) \ / \ N_0(t) \approx Bin(N_0(t), s_0)$

$\Downarrow$

$N_1(t+1) \approx Poisson(N_1(t)f_1s_0 + N_2(t)f_2s_0)$

Overall model: a branching process

$N_1(t+1) \approx Poisson(N_1(t)f_1s_0 + N_2(t)f_2s_0)$

$N_2(t+1) \approx Bin(N_1(t),s_1) + Bin(N_2(t),s_2)$

Year t newborn Year t+1

$N_1 \xrightarrow{f_1}$ newborn

$N_2 \xrightarrow{f_2}$ newborn

newborn $\xrightarrow{s_0} N_1$

$N_1 \xrightarrow{s_1} N_2$

$N_2 \xrightarrow{s_2} N_2$

An even simpler example of branching process

$N(t+1) \approx \text{Poisson}((s_0 f + s_1)\, N(t)) = \text{Poisson}(\lambda N(t))$

Demographic vs Environmental stochasticity

Demographic
$N(t+1) \approx \text{Poisson}((pf+q)\, N(t)) = \text{Poisson}(\lambda N(t))$
$E(N(t+1) / N(t)) = \lambda N(t) \quad var(N(t+1) / N(t)) = \lambda N(t)$
More generally $var(N(t+1) / N(t)) = \alpha N(t)$

Demographic vs Environmental stochasticity

Demographic
$N(t+1) \approx \text{Poisson}((pf+q)\, N(t)) = \text{Poisson}(\lambda N(t))$
$E(N(t+1) / N(t)) = \lambda N(t) \quad var(N(t+1) / N(t)) = \lambda N(t)$
More generally $var(N(t+1) / N(t)) = \alpha N(t)$

Environmental
$N(t+1) / N(t) \approx \Lambda N(t)$, Λ random variable, $E(\Lambda)=\lambda$
$E(N(t+1) / N(t)) = \lambda N(t) \quad var(N(t+1) / N(t)) = \sigma^2 N(t)^2$
More generally $var(N(t+1) / N(t)) = \alpha N(t)^2$

Demographic vs Environmental stochasticity

Demographic
$N(t+1) \approx \text{Poisson}((pf+q)\, N(t)) = \text{Poisson}(\lambda N(t))$
$E(N(t+1) / N(t)) = \lambda N(t) \quad var(N(t+1) / N(t)) = \lambda N(t)$
More generally $var(N(t+1) / N(t)) = \alpha N(t)$

Environmental
$N(t+1) / N(t) \approx \Lambda N(t)$, Λ random variable, $E(\Lambda)=\lambda$
$E(N(t+1) / N(t)) = \lambda N(t) \quad var(N(t+1) / N(t)) = \sigma^2 N(t)^2$
More generally $var(N(t+1) / N(t)) = \alpha N(t)^2$

	$var(N(t+1) / N(t))$
Demographic	$\alpha N(t)$
Environmental	$\alpha N(t)^2$

Demographic vs Environmental stochasticity

	$var(N(t+1) / N(t))$
Demographic	$\alpha N(t)$
Environmental	$\alpha N(t)^2$

- Environmental Stochasticity prevails over Demographic Stochasticity in large populations
- Demographic Stochasticity prevails over Environmental Stochasticity in small populations
- Demographic Stochasticity may be non negligible in large, multistate populations (small number of individuals in some states)

Demographic stochasticity and extinction

- In Branching Processes, population size is an integer
- Extinction is unambiguously defined as reaching a population size equal to 0
- … or a vector population size equal to (0, 0, …, 0) in the case of a structured population
- Many formal results in mathematical literature
- e.g., extinction certain iff λ <=1
- Simulation straightforward in ULM

Demographic stochasticity in ULM

"rel" = recurrence relationships

```
{ Swallow with demographic stochasticity
defmod swallowDS(2)
rel : rn1,rn2

{ relation for n1
defrel rn1
n1 = poisson(n1*s0*(f1+f2))

{ relation for n2
defrel rn2
n2 = binomf(n1,s1) + binomf(n2,s2)
```

Demographic stochasticity in ULM

declaring variables and parameters

```
{ initial numbers
defvar n1 = 10

defvar n2 = 10

{ total number
defvar n = n1 + n2

{ 1st year survival prob.
defvar s0 = 0.3
```

```
{ 2nd year survival prob.
defvar s1 = 0.50

{ After2nd year survival prob.
defvar s2= 0.65

{ subadult female fecundity
defvar f1 = 3.0/2

{ adult female fecundity
defvar f2 = 6.0/2
```

Demographic stochasticity in ULM

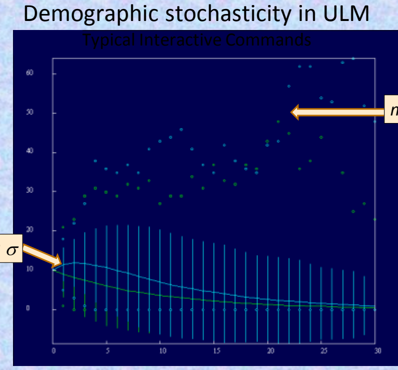
Typical Interactive Commands

```
graph

Montecarlo 30 1000
```

In the graph window, click on
select "Min Max", "2 sigma"

Time steps   Replicates

Demographic stochasticity in ULM



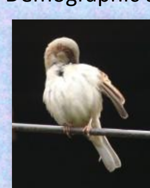
Demographic stochasticity and Density-dependence

If λ > 1, asymptotically P(N(t)=0) + P(N(t)=∞) = 1, i.e. the population either goes extinct or escapes to ∞

If DD is added to stabilize the population, then, asymptotically P(N(t)=∞) = 0

Hence asymptotically P(N(t)=0) = 1, i.e. extinction is certain

Demographic stochasticity and Density-dependence



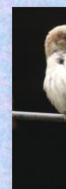
```
defmod sparrow(1)
rel : rn1

{ relation for n1
defrel rn1
n = poisson(n*s0*f)+binomf(n,s1)

{ initial numbers
defvar n = 10

{ FY survival probability
defvar s0 = 0.2*exp(-0.01*n)

{ AFY survival probability
defvar s1 = 0.5*exp(-0.01*n)

{ female fecundity
defvar f = 6.0/2
```

Demographic stochasticity and Density-dependence

```
defmod sparrow(1)
rel : rn1

{ relation for n1
defrel rn1
n = poisson(n*s0*f)+binomf(n,s1)

{ initial numbers
defvar n = 10

{ FY survival probability
defvar s0 = 0.2*exp(-0.01*n)

{ AFY survival probability
defvar s1 = 0.5*exp(-0.01*n)

{ female fecundity
defvar f = 6.0/2
```

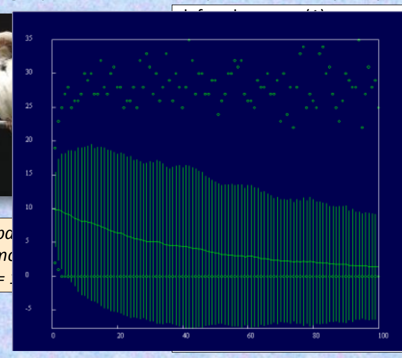
***Low** baseline survival and moderate DD*
λ_{max} = 1.1

Demographic stochasticity and Density-dependence

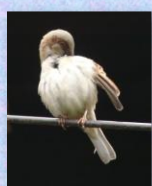


Low ba
and mo
λ_{max} =

s1)



Demographic stochasticity and Density-dependence



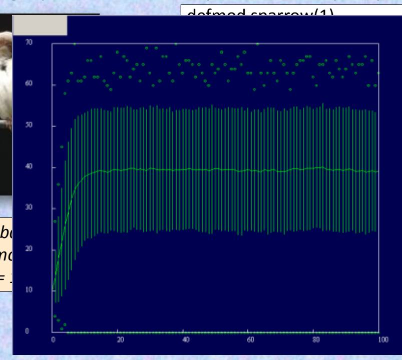
```
defmod sparrow(1)
rel : rn1

{ relation for n1
defrel rn1
n = poisson(n*s0*f)+binomf(n,s1)

{ initial numbers
defvar n = 10

{ FY survival probability
defvar s0 = 0.3*exp(-0.01*n)

{ AFY survival probability
defvar s1 = 0.6*exp(-0.01*n)

{ female fecundity
defvar f = 6.0/2
```

***High** baseline survival and moderate DD*
λ_{max} = 1.5

Demographic stochasticity and Density-dependence

High b
and mo
λ_{max} =

s1)



Demographic stochasticity and Density-dependence

If DD is added to stabilize the population, extinction is certain

- Population size stabilizes conditional on non extinction (**Quasi-stationary distribution, QSD**)
- Pr(extinction in one time step) becomes constant
- Time to extinction: geometric distribution
- Pr(Extinction over finite time window) often negligible

The QSD concept leads to a continuum from decreasing population doomed to close extinction to stable DD populations with negligible risk of extinction

UF UNIVERSITY of FLORIDA