Closed Capture-Recapture Models
2 Sample Model

Outline:

- Model description/ data structure
- Encounter history
- Estimators
- Assumptions and study design

Basics of CMR

- Basic principle: $$\hat{N} = \frac{n}{\hat{p}}$$
- If sample of 50 and encounter rate is 0.5
- Marked individuals

Capture & Marking

Natural marks

Marking

- Should have minimal impact on
  - Survival
  - Behavior

- Mark should be reliable (e.g., tag loss)

Capture-Recapture Models for Closed Populations

- Closure: no changes in numbers or identities of animals between sampling periods
  - Demographic closure
  - Geographic closure

- Studies typically conducted over short time periods
- Estimation focus is on population size, N

2-Sample Capture-Recapture

- Catch animals at sample period 1, mark, and release back into population

- Recapture animals at sample 2, recording number with and without marks

2-Sampling Occasions LP

$$\hat{N} = \frac{n_1 n_2}{m_2} \qquad \frac{n_1}{N} = \frac{m_2}{n_2}$$

N-hat: estimate of the total no of indiv. in the population

n_1: no caught on the 1st occasion

n_2: no caught on the 2nd occasion

m_2: no of animals recaptured on the second occasion

Batch marking is possible in this case

Frederick C. Lincoln (1930)

$$\frac{\text{(\# banded ducks shot)}}{\text{(\# ducks banded)}} = \frac{\text{(\# ducks shot)}}{\text{(\# ducks)}}$$

$$\text{Thus: } \underbrace{\text{Estimated \# ducks}}_{\textbf{N-hat}} = \frac{n_2\,\text{(\# ducks shot)}\ \overset{n_1}{\text{(\# ducks banded)}}}{m_2\ \ \text{(\# banded ducks shot)}}$$

Chapman's estimator

$$\hat{N} = \frac{(n_1 + 1)(n_2 + 1)}{(m_2 + 1)} \ -1$$

N-hat: estimate of the total no of indiv. in the pop
n_1: no caught on the 1st occasion
n_2: no caught on the 2nd occasion
m_2: no of animals recaptured on the second occ.

$$\hat{\mathrm{v}}\mathrm{ar}(\hat{N}) = \frac{(n_1 + 1)(n_2 + 1)(n_1 - m_2)(n_2 - m_2)}{(m_2 + 1)^2 (m_2 + 2)}$$

Encounter History Data

Capture History	Model
10	$p_1(1-p_2)$
01	$(1-p_1)p_2$
11	$p_1 p_2$
00	$(1-p_1)\ (1-p_2)$

Row vector of 1's (indicating capture) and 0's (indicating no capture)

Statistics and Intuitive estimation

x_{ij} = number of animals with history *ij*

$$\hat{p}_1 = \frac{\overset{m_2}{x_{11}}}{\underbrace{x_{01} + x_{11}}_{n_2}} \qquad \hat{p}_2 = \frac{x_{11}}{\underbrace{x_{10} + x_{11}}_{n_1}}$$

$$\hat{N} = \frac{x_{10} + x_{11}}{\hat{p}_1} = \frac{x_{01} + x_{11}}{\hat{p}_2}$$

$$= \frac{\overbrace{(x_{01} + x_{11})}^{n_2}\overbrace{(x_{10} + x_{11})}^{n_1}}{x_{11}} \text{ = LP estimator}$$

Full Likelihood approach

$$P(n_1, n_2, m_2 \mid N, p_1, p_2) = \frac{N!}{m_2!(n_1 - m_1)!(n_2 - m_2)!(N - r)!}$$
$$\times (p_1 p_2)^{m_2} [p_1(1 - p_2)]^{(n_1 - m_2)} [(1 - p_1)p_2]^{(n_2 - m_2)} [(1 - p_1)(1 - p_2)]^{(N - r)}$$

$$\mathcal{L}(N, \mathbf{p} \mid \text{data}) \propto \frac{N!}{(N - M_{t+1})!} \prod_h P[h]^{n_h} \cdot P[\text{not encountered}]^{N - M_{t+1}}$$

where $\mathbf{p}$ is the vector of encounter probability parameters, M_{t+1} is the number of unique animals marked, and n_h is the number (frequency) of individuals with encounter history h.

L-P Assumption Violations: Closure

- Only losses between 1 and 2
  - Equal probability of loss for marked and unmarked: no bias in $\hat{N}_1$, i.e., LP estimator estimates abundance at time 1
  - Only marked animals are lost (handling effect): LP estimator positively biased
- Only gains between 1 and 2: no bias in $\hat{N}_2$
- Both gains and losses occur between 1 and 2: positive bias in LP estimator for both N_1 and N_2

L-P Assumption Violations: Equal Capture Probability

- Heterogeneous capture probability among individuals: high-*p* animals in initial sample more likely to be recaptured so $\hat{p}$ is too large and $\hat{N}$ is biased low
- Trap response for animals caught in 1
  - Trap happy response: $\hat{N}$ is negatively biased
  - Trap-shy response: $\hat{N}$ is positively biased

Heterogeneity of Capture

Colorful vs cryptic (male vs female)
Big vs Small
Fast vs Slow (e.g. pregnant female)
Smart vs Sucker…

Collect info for possible stratification

Trapping Effect

"Trap shy" (e.g. because stress manipulation)
P (capture an animal already captured) <
P (Animal never captured)

"Trap happy" (e.g. because food in trap)
P(capture an animal already captured) >
P(Animal never captured)

Tip: Use different method for "recapture"
Or >2 sampling occasions with indiv. marks

L-P Assumption Violations: No Tag Loss

- Tag loss leads to underestimation of capture probability and positive bias for $\hat{N}$
- Estimate tag loss (e.g., with double-marking study)
- If P(tag retention between 1 and 2) = θ, then abundance can be estimated as:

$$\hat{N} = \hat{N}_{LP} / \hat{\theta}$$

2-Sample Design Issues
Closure

- Relatively short interval between samples (depends on organism)

- Try to minimize trap and handling mortality
  - Check trap early (cold- heat stress); feeding (starvation)
  - If die first occasion $n_1' = n_1 - d$
  - N': pop. after sampling; and N'+d is presampling pop.

  - Avoid migration periods

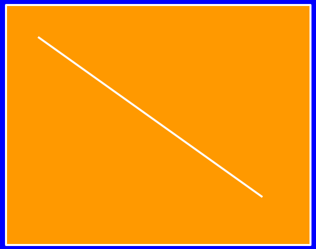
2-Sample Design Issues
Equal capture probability

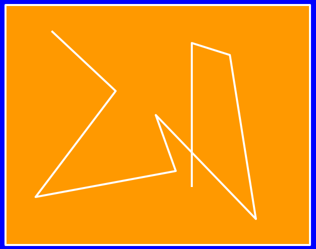
- Collect ancillary information (e.g., sex, age, size) for possible stratification
- Trap shyness: minimize handling time
- Use different capture methods for the 2 samples
– Different methods for initial “capture” and “recapture”
– This is because capture probabilities may differ between samples
    - Rabbit example in Oregon, batch marking (picric), and resight (drive count)

- Model behavioral response K>2

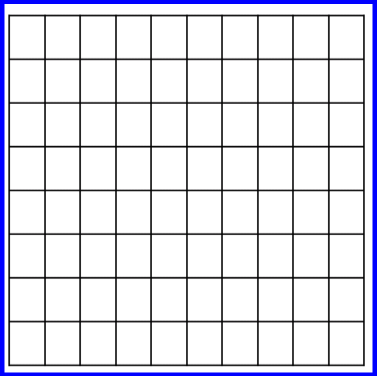
Sample placement

- Trap lines
  - Poor dispersal of traps

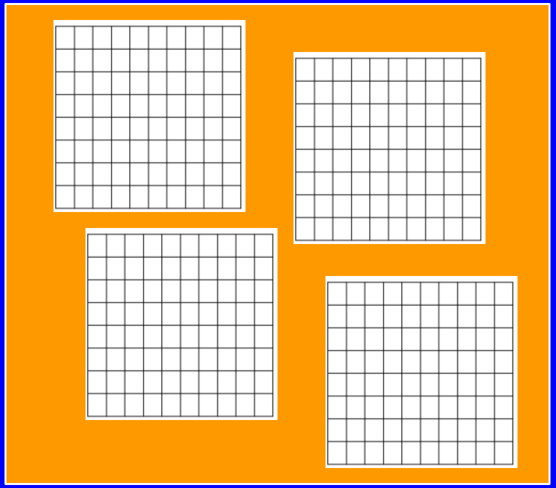




- Uniform grid



- Replicated subgrids
  - N-hat for each replicate, and empirical variance



- Stratification

Sample size

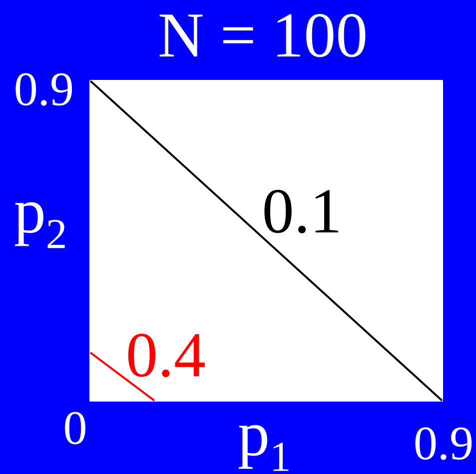
- Precision of LP estimator will depend on n_1, n_2, and m_2
  - Which may depend on effort (e.g., # traps)
  - Actual abundance
  - Effectiveness of capture methods…
- Given N, p_1 and p_2 can determine

$$CV(\hat{N}) = \frac{\sqrt{\hat{\text{var}}(\hat{N})}}{\hat{N}}$$

- Pilot study for N and capture prob.

$$\hat{\text{var}}(\hat{N}) = \frac{(n_1+1)(n_2+1)(n_1-m_2)(n_2-m_2)}{(m_2+1)^2(m_2+2)}$$

$$n_1 = Np_1; \quad n_2 = Np_2; \quad m_2 = Np_1p_2$$



Conroy and Carroll (2010)

Take home points

- LP estimator
- Assumptions
  - Closure
  - Heterogeneity
- Design issues
  - How to meet assumptions
- Mark resight models