Bayesian Model Updating

UF 2015

Outline (20 min)

- Multiple hypotheses
- Update of model weight
- Example

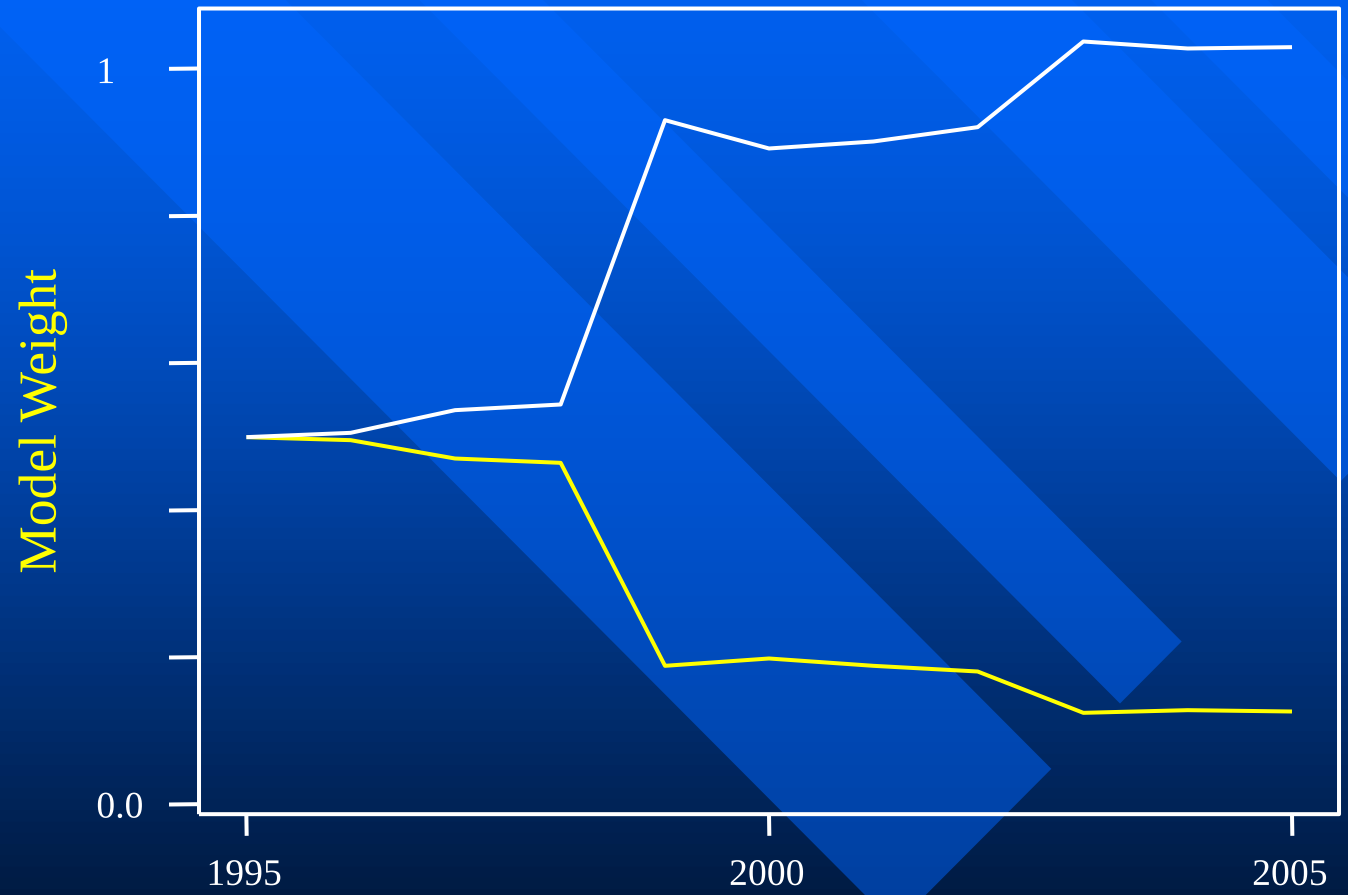
Multiple-Hypothesis Approach to Science

- Develop set of competing hypotheses
- Develop/derive prior probabilities associated with these hypotheses
- Use associated models to deduce predictions
- Carry out suitable test
- Compare test results with predictions
- Based on comparison, compute new probabilities for the hypotheses

For a given model set

- Weights assigned to each model add to 1.0 (thus relative credibility)
- Models with higher weight have greater credibility and will have more influence over future management decisions
- If a robust predictive model is in the set its weight should go to 1.0 over time.

Update of model weight over time



Initial Weight Values

- Option 1 – set subjectively
  - Politically (should be avoided!)
  - Based on expert opinion
- Option 2 – use historical data
  - AIC weights (Burnham and Anderson 1998),
  - Pick previous date, start with equal weights, and update to present time.

Weights updated as function of:

- The current weight (*prior probability*)
- New information (i.e., the difference between model predictions and what actually occurs, based on monitoring results).
- The new weight is called a *posterior probability*

Updating Model Probabilities: Bayes' Theorem

$$p_{t+1}(m_i \mid dat_{t+1}) = \frac{p_t(m_i) \times P(dat_{t+1} \mid m_i)}{\sum_{i=1}^{n} p_t(m_i) \times P(dat_{t+1} \mid m_i)}$$

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Relationship to Likelihood Theory

- $P(dat_{t+1} \mid m_i)$ is often obtained as the likelihood function evaluated at the maximum likelihood estimates

Process furthers learning when

- A good approximating model is in the model set (i.e., a model that predicts well across the state space).
- Predictions from each model fairly represent the idea that generated them.

Model predictions should:

- Be unbiased under the ecological hypothesis they represent.
  - Bias could change direction of weight changes and lead to throwing out hypothesis erroneously.
- Include all pertinent uncertainties
  - Parametric uncertainty – sampling variation due to estimation.

Multiple-Hypothesis Approach to Science

- Make better use of previous research (e.g., Descartes 1644)
- Use literature reviews to compute prior probabilities associated with competing hypotheses (meta-analytic approach)

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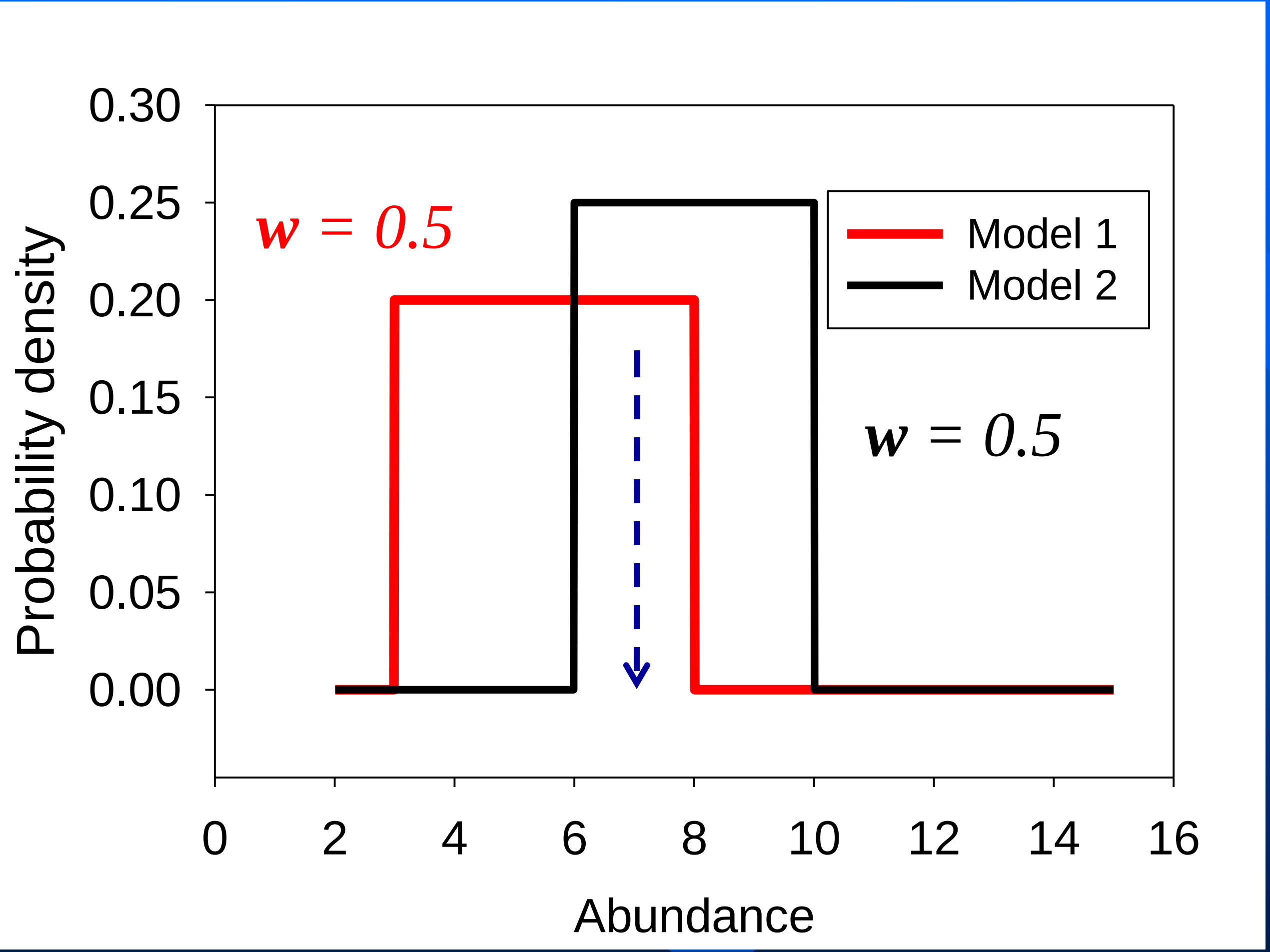
Generic example – 2 models

- Assume equal priors
  - each model gets weight of 0.50
- Compute posteriors (i.e., update weights) based on comparison of predicted *observed* state (e.g., population size) with resulting *observed* state.

Example – 2 models Uniform distribution



Bayes' Formula

$$p_{t+1}(m_i \mid dat_{t+1}) = \frac{p_t(m_i) \times P(dat_{t+1} \mid m_i)}{\sum_{i=1}^{n} p_t(m_i) \times P(dat_{t+1} \mid m_i)}$$

$$p_{t+1}(m_1 \mid dat_{t+1}) =$$

$$\frac{0.5 * 0.2}{0.5 * 0.2 \ + \ 0.5 * 0.25} = 0.44$$

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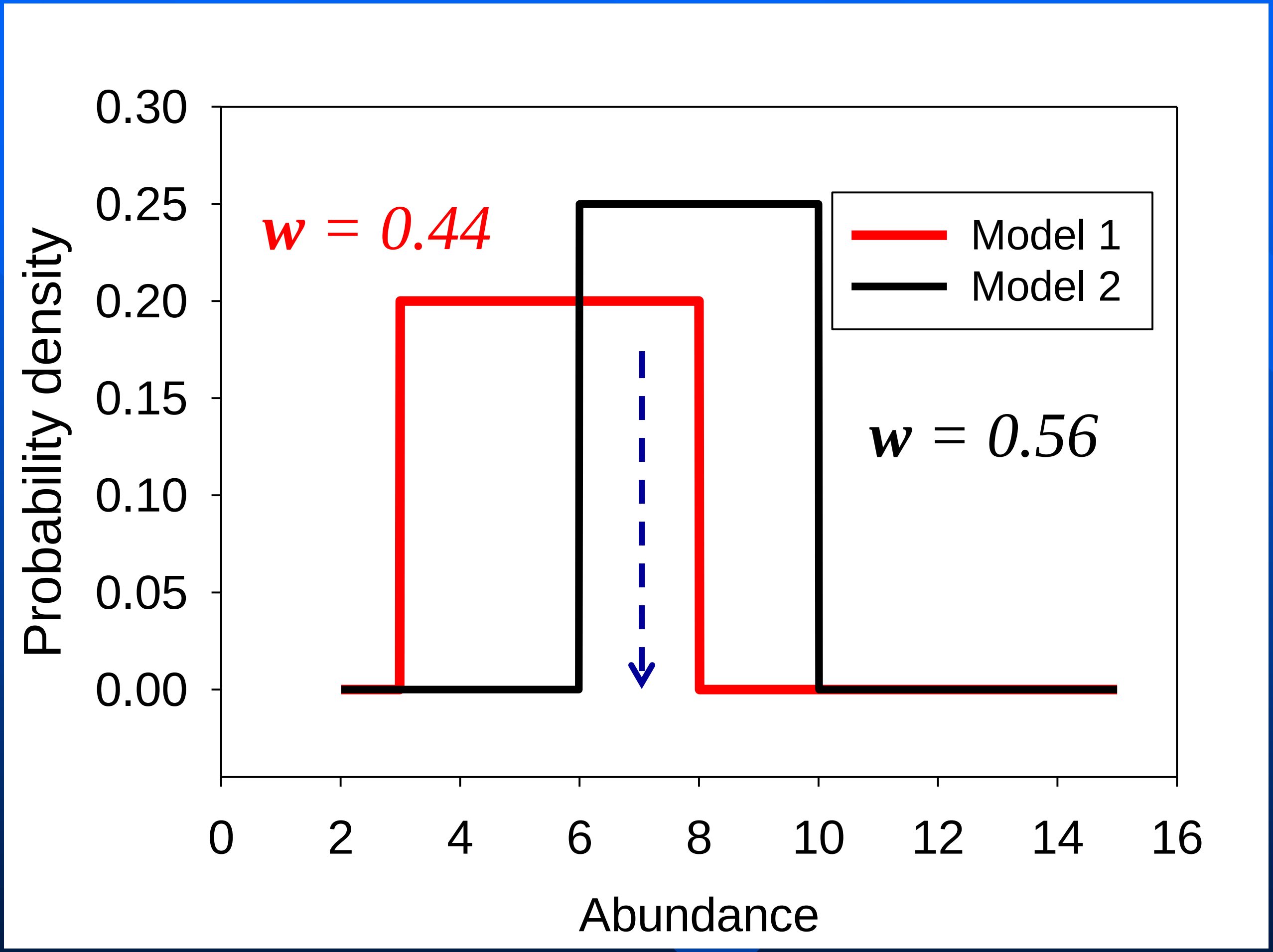
Example – 2 models
Uniform distribution



Take home points

- Purpose of Bayes updating
  - Multiple hypotheses approach to science
  - Identify model that represents best approximation
- How it works (see equation)

$$p_{t+1}(m_i \mid dat_{t+1}) = \frac{p_t(m_i) \times P(dat_{t+1} \mid m_i)}{\sum_{i=1}^{n} p_t(m_i) \times P(dat_{t+1} \mid m_i)}$$

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