# II. Capture-Recapture models with fixed covariates

# TYPES OF CR MODELS

**Now:**

- Model Mb – Behavioral response model. Animals can be "trap happy" or "trap shy"
- Model Mt – Time specific encounter probability.
- Model Mh – models of "individual heterogeneity" in p. Each individual has it's own encounter probability p[i], and this is a realization of some random variable.

**Later:**

- Models for stratified populations (multi-session models)
- Model Mx – individual covariate models
  - SCR models are just a type of Model Mx.

# Model Mb

- Behavioral response model
- Trap happiness, or trap shyness
- p depends on whether captured previously, p(before), p(after)
- Persistent (or permanent) vs. Transient (or ephemeral)

# Analysis of model Mb

Can be regarded as a type of logit model with a covariate being "previous capture"

x(i,k) = 1 if individual i was captured prior to sample occasion k

logit(p[i,k]) = alpha0 + alpha1*x(i,k)

Or

p[i,k] = p1*(1-x(i,k)) + p2*x(i,k)

**Parameterizing intercepts of models in terms of probability scale parameters is preferred – better mixing.**

# Model Mb: Implementation in BUGS

- Because p depends on individual AND sample occasion, the model must be formulated in terms of the binary observations y[i,k]

- **R work session**

# COMMENTS

- Behavioral response models are important in CR studies because traps are often baited. Especially with new types of passive trapping methods (hair snares, camera traps) there is usually a +ve behavioral response.
- Parameterization can be really important. Try to use probability scale parameters.
- Persistent vs. ephemeral or transient behavioral response.
- Behavioral response models are models with a fixed covariate. Other types of models might be “occasion effects” (date or environmental conditions). No new considerations.

# Model Mt

- Time-specificity: p changes over sampling occasions

    “Trend model”

    full time-specificity

- R work session

# Key point: toward model Mx

- For these models (Mb and Mt) the covariate is **known** for all individuals whether captured or not.
- Models are simply zero-inflated Bernoulli or binomial regression models
- For many types of covariates this is not the case. E.g., consider "sex" == we observe this for captured individuals (sometimes not all) but we don't know this for uncaptured individuals.
- Later we'll talk about these "individual covariate" models.

# BAYESIAN MODEL SELECTION

We can fit a bunch of models, how do we choose among them?

- Method 1: look at posterior mass, if not in the vicinity of 0, then "significant"

- Method 2: DIC – deviance information criterion. Calculation of "effective degrees of freedom" is problematic.  Posterior deviance also seems unstable. Jury is still out on DIC but it is widely used.

- Method 3: Kuo and Mallick (1998) indicator variables approach.

# DIC

Spiegelhalter, D. J., Best, N. G., Carlin, B. P., & Van Der Linde, A. (2002). Bayesian measures of model complexity and fit. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, *64*(4), 583-639.

DIC = -2*loglikelihood + pD

- pD = penalty based on "effective number of parameters"
- pD = posterior mean deviance – deviance evaluated at the posterior mean of parameters (WinBUGS does this differently)

Think about it like AIC

Use with caution

# DIC has issues

Lunn et al. ("The BUGS book", 2012) noted problems with DIC in the way pD is calculated

1. pD is not invariant to reparameterisation, in the sense that if the model is rewritten into an equivalent form but in terms of a function g(theta), then a different pD may arise. This can lead to misleading values in some circumstances and even to negative values of pD.

2. An inability to calculate pD when theta contains a categorical parameter, since the posterior mean is not then meaningful. This renders the measure inapplicable to mixture models [....].

The latent variables z are exactly such a categorical parameter.

# Deviance and DIC for fort drum black bear models

| | dev | sd | pD | DIC |
|---|---|---|---|---|
| fit0 | 489.4968 | 11.49556 | 66.07470 | 555.5715 |
| fit.Mbv1 | 558.5401 | 45.46447 | 1025.53174 | 1584.0719 |
| fit.Mbv2 | 544.9618 | 39.96822 | 795.18595 | 1340.1477 |
| fit.Mt1 | 487.9645 | 11.46732 | 65.73807 | 553.7026 |
| fit.Mt2 | 477.7443 | 11.59752 | 67.25001 | 544.9943 |
| fit.Mtb | 562.0742 | 50.01249 | 1228.34109 | 1790.4153 |

DIC seems very sensitive to the number of individuals that were not encountered.

# INDICATOR VARIABLES

- Kuo and Mallick (1998) idea is to expand the model to include binary variables like this:

  logit(p[i,t]) = alpha0 + w*alpha1*x[i,t]

- Put a prior distribution on the binary indicator variable w

  w ~ Bern(.5)

- Estimation of w is equivalent to deciding if x[i,t] should be in the model

# FORT DRUM MODEL

```
w[1] ~ dbern(.5)
w[2] ~ dbern(.5)

for (i in 1:M){
   z[i]~dbern(psi)
   for(k in 1:K){
     logit(p[i,k]) <- alpha0+ w[1]*beta[occasion[i,k]]+
w[2]*alpha1*prevcap[i,k]
     tmp[i,k]<-p[i,k]*z[i]
     y[i,k]~dbin(tmp[i,k],1)
      }
     }
```

# Posterior model probabilities

```
> model<- paste(fit.Mtb.ms$sims.list$w[,1],
               fit.Mtb.ms$sims.list$w[,2])
> table(model)
model
   0 0    0 1    1 0    1 1
 13649 346245     15     91
>
>
```

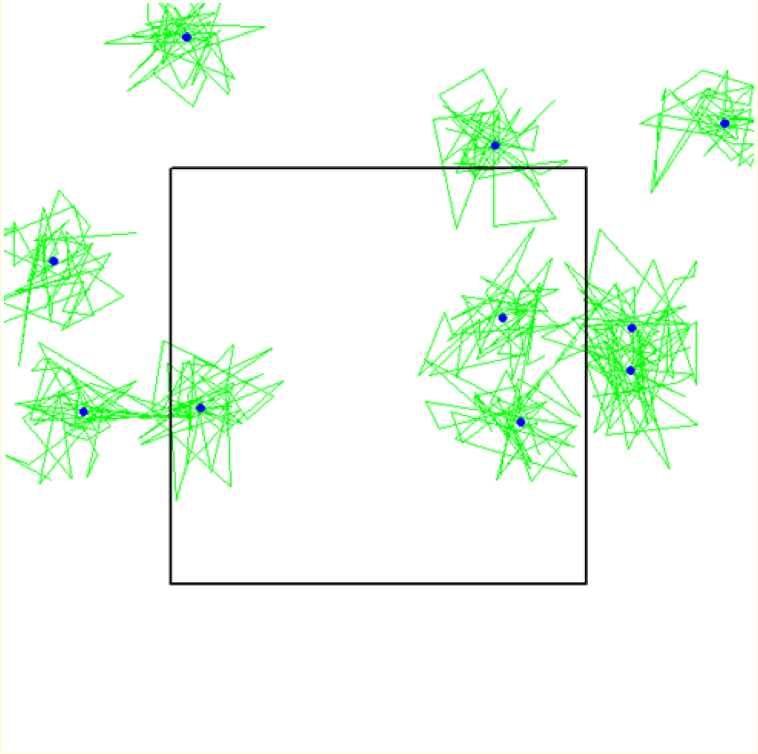
Pr(model = “0 1”) = 0.9618

# BAYESIAN MODEL SELECTION SUMMARY

- With fixed effects we can compute posterior model probabilities using the model indicator variables
- DIC – don’t use this (but some good calibration studies might be useful)
- If you have a small number of factors, no reason you couldn’t use judgment based on posterior mass.

# Temporary emigration

- A specific type of non-closure that is highly motivation to SCR models.
- Individuals are not always available to be captured because they are “not in the study area”.
- Individuals on the edge of the study area have higher temporary emigration rates.



# Consequences of non-closure

- Temporary emigration -> heterogeneity in p
- Estimates of encounter probability, p, are biased HIGH and therefore estimates of N are biased LOW.

# Model Mh

- Movement about the edge of the trap array or variable exposure to trapping induces heterogeneity

- So people have tried to explain that using "model Mh" (Karanth and Nichols papers)

- Maybe not such a good idea according to Bill Link (see 2003 Biometrics paper). The idea here is that for different types of model Mh, it is impossible to choose among them, but they produce wildly different estimates of N.

- **Better to try to explain heterogeneity explicitly. Model Mx is an attempt at doing that.  SCR models are a better attempt.**

# Model Mh

- Model Mh is a binomial encounter model just like every other CR model:

  y[i] ~ Binomial(K, p[i])

- But now we have p depending on individual, p[i], and we regard p[i] as a random effect:

  p[i] ~ g(theta)

- g(theta) is some probability distribution

# FLAVORS OF MODEL MH

A large number of Models Mh have been proposed depending on the form of g(theta)

- **Logit-normal model:**
  p[i] ~ Normal(mu, sigma^2)   (Coull and Agresti 1999 et al)

- **Beta model:**
  p[i] ~ beta(a, b)   (Burnham's PhD Dissertation,
  Dorazio and Royle (2003) )
- **"finite mixtures"**
  p[i] ~ latent classes   (Norris and Pollock '96
  Pledger 2000, et al)

# Fitting model Mh

- Finite-mixture model. This is really a key model because SCR models look like a type of finite mixture model when we use a discrete state-space.  Also the package "secr" fits this type of model Mh.

- Estimates of N are really sensitive to which model we choose for heterogeneity!!!  (Link 2003 Biometrics)

- This is because they have radically different behaviors as p -> 0 and most real data sets don't provide much information about that behavior.

# Finite Mixtures

- Finite mixture are also called "latent class models". The idea is some parameter depends on some unobserved (latent) class variable.
- For 2 groups, p[1] and p[2] are the parameters and there is a latent variable "class" such that Pr(class[i] = 1) = f
- **Parameters are p[1], p[2] and f**
- BUGS-like notation:

```
y[i] ~ dbin(p[class[i]], K)

   class[i] ~ dcat( probs[] )

   class[i] is MISSING for all individuals in the population
```

- **R** work session: `closed_models_part3.R`

# MORE ON THE CATEGORICAL DISTRIBUTION

- Multinomial trial: a vector of 0's with a single 1:

y= (0,0,0,1,0,0)

~ multinomial(n=1 ; probs[])

- Same as

y = 4 (the position of the 1)

~ categorical(probs[])

# SUMMARY OF PART II

- Fixed covariates are easy to handle in CR models

- We use indicator variables to do Bayesian model selection in BUGS

- Model Mh == “latent covariate” to explain heterogeneity
  - Finite mixture model is widely used, easy to implement in BUGS
  - Practical utility of Model Mh has been called into question

(Extreme sensitivity to class of models (Link 2003))
  - Nevertheless historical context is interesting and important
  - “secr” fits a type of Model Mh based on the finite mixture models (latent class)

- **Models with class structure are equivalent to Model Mh but with some observed classes (sex, year, site, age class, etc..)**