Non-Spatial Closed Population Capture-Recapture Models for Estimating N

Outline

Part I

- Basic design and data structure
- "Model M0" – likelihood analysis
- Bayesian analysis of Model M0 – Data augmentation
  - Converts Model M0 to "site occupancy" model
  - Model M0 in BUGS

Part II

- Model Mb (behavioral response), Model Mt and related….
- Model selection in BUGS

Transition to spatial capture-recapture

Part III

- Model Mh
- Discrete class models (stratified populations)

Part IV

- Model Mx (individual Covariates)

Closed populations

- Simplest conceptual model of a population is that it is "closed". This has two components:
  - (a) Demographic closure. No recruitment and no mortality
  - (b) Geographic closure. Animals don't leave the population (no emigration) or enter the population (no immigration).
- Model is that of a fish bowl or other spatially constrained population over a short period of time
- Closure cannot possibly hold in real populations.

Sampling a closed population

- Sampling model: individuals are randomly selected from a population with probability p == *per sample encounter or capture or detection probability*

- Conceptually this is a Bernoulli sampling model: whether each individual appears in the sample is a "coin toss":

  y[i] ~ Bernoulli(p)   for i = 1, 2, … N

  N = population size

- CR models: many different ways that p can vary (later…)

Closed populations: data structure

- We estimate p by obtaining replicate samples from the population. Let K = number of replicate samples. Individuals are released after each sample, may be recaptured.

- Produces individual encounter histories (**n x K** matrix)

	sample 1	sample 2	sample 3	TOTAL
Individual 1	1	0	1	$y_1 = 2$
Individual 2	0	0	1	$y_2 = 1$
Individual 3	1	1	1	$y_3 = 3$
etc..				
Individual n	0	1	0	$y_n = 1$

Closed population models

- We need to estimate p in order to estimate N

Under random sampling:

n ~ Binomial(N, $\bar{p}$)

This is the probability that an individual appears in the sample over the K occasions.

$$\bar{p} = 1 - (1 - p)^K$$

The heuristic estimator of N:

E(n) =$\bar{p}$*N

N = n/$\bar{p}$

("moment estimator", equate the 1st moment of our statistic n to its expected value and solve)

Closed population models

- Estimating p is really important!
- How do we estimate p?
- Dozens of models have been proposed that differ mainly in how p varies by individuals, time, etc..

Otis et al (1978) characterization of closed models

- The standard models:
  - M0 = “the null model”, p is constant in all dimensions
  - Mt = p is a function of sample occasion , p(t)
  - Mb = behavioral response model. Trap happiness or shyness
  - Mh = individual heterogeneity
  - Mbt = time + behavior, or time*behavior
  - Mbh, Mth, Mbth

- See Kery and Schaub (2012) Ch. 6 for how to do all of these in WinBUGS/JAGS

MODEL M0

Model M0 is a common point of reference in capture-recapture. It consists of the following assumptions:

- Encounter probability, p, constant for all sample occasions and all individuals

- Then, encounter observations are Bernoulli random variables (just coin flips) and the individual frequencies are binomial:

y[i,k] ~ Bernoulli(p) for all i=1,2,..,N and k=1,2,...,K

-- same as --

y[i] ~ Binomial(K, p) for all i=1,2,...,N

ANALYSIS OF MODEL M0

- Looks like binomial GLM, logistic regression, etc..
- Key technical issue: unlike a typical GLM, N, the size of some ideal data set, is unknown
- 3 things we have to talk about:
  - "conditional likelihood"
  - "full likelihood"
  - "data augmentation"

The binomial model and the likelihood

- Under model M0 assumptions, encounter frequencies are binomial:

  y_i ~ Binomial(K, p) for all i=1,2,…,N

- But N is not known, we only observe y_i IF $y_i > 0$. i.e., the observed data have a "**zero-truncated binomial**" distribution

  $f(y) = Bin(y; K, p)/(1-(1-p)^K)$

- **This is the basis of the "conditional likelihood" for estimating parameters of closed population models.**

- This is called the "conditional likelihood" because it is "conditional on capture", i.e., conditional on y>0, or "conditional on n"

Conditional likelihood in R

```
lik0.cond<-function(parms){
     p<-  plogis(parms[1])
     pcap<- 1-(1-p)^K
     part1<- sum(log(dbinom(y,K,p)/pcap))
     -1*(part1)
}
```

The full likelihood

- But n is also part of the observable data. What is the distribution of n?

$$n \sim \text{Bin}(N, 1-(1-p)^K)$$

- So the "joint likelihood" or "full likelihood" is the product of the previous bit (the conditional likelihood) and this bit for n:

Full likelihood = [**conditional likelihood**] * **Bin(N, 1-(1-p)^K)**
= binomial likelihood with combinatorial term

This is called the "full likelihood" "joint likelihood" "unconditional likelihood" because it has N in it.

The full likelihood as an R function

```
lik0<-function(parms){
 p<-  plogis(parms[1])
 n0<-  exp(parms[2])
 N <-nind + n0
 part1<- sum(log(dbinom(y,K,p)))
 part2<-lgamma(N+1) -
      lgamma(n0+1) + n0*log(dbinom(0,K,p))
-1*(part1 + part2)
}
```

```
In R, lgamma(N+1) = log(factorial(N))
```

Simulate some data and obtain the MLE

R work session

Fort Drum bear data

Hair snare study

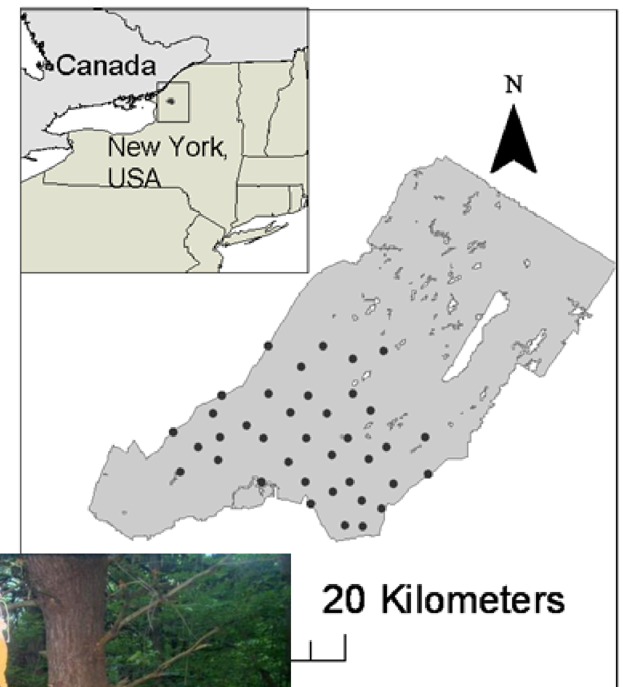
J = 38 hair snares

K = 8 weeks of sampling

n = 47 individuals captured

Load the data:

```
library(scrbook)
data(beardata)
```



Fort Drum bear data

```
> library(scrbook)
> data(beardata)

> str(beardata)
List of 4
 $ trapmat  :'data.frame':      38 obs. of  2 variables:
  ..$ V1: num [1:38] 448 439 439 442 442 ...
  ..$ V2: num [1:38] 4886 4881 4879 4884 4881 ...
 $ bearArray: num [1:47, 1:38, 1:8] 0 0 0 0 0 0 0 0 0 0 ...
 $ flat     : num [1:151, 1:4] 1 1 1 1 1 1 1 1 1 1 ...
  ..- attr(*, "dimnames")=List of 2
  .. ..$ : NULL
  .. ..$ : chr [1:4] "Session" "ID" "Occasion" "trapID"
 $ sex      : num [1:47] 1 1 2 1 1 1 1 2 1 2 ...
```

BASIC DATA FORMATTING

- In practice we have too much data for ordinary capture-recapture models
  - Individuals can be captured at > 1 trap during a sample occasion
- Therefore we have to summarize the data (i.e., throw some of it out)
- A typical encounter data file (EDF) has 3 pieces of information
  - Individual captured
  - Trap of capture
  - Occasion of capture

BASIC DATA FORMATTING

- A typical encounter data file (EDF) has 3 pieces of information
  - Individual captured
  - Trap of capture
  - Occasion of capture

- It is convenient to organize this into a 3-dimensional array: **individuals x traps x occasions**

- In order to fit ordinary CR models we need to reduce this to a 2-dimensional matrix: **individuals x occasions**

- Lets do this for the Fort Drum bear data

THIS IS REALLY IMPORTANT!

- `bearArray` = the encounter data, is a 3-d array.....
  - Have to summarize over traps to fit ordinary closed models
  - Multiple captures in a sample occasion have no meaning

```
y <- beardata$bearArray
y <- apply(bearArray,c(1,3),sum)
y[y>1] <- 1          # multiple captures are redundant.
y.summed <- apply(y,1,sum)# total encounters out of K
```

- We model either the matrix `y` or the vector `y.summed`

Fort Drum bear data

R work session

Summary so far

Model M0

Its essence is a simple binomial model, just like logistic regression

- Conditional likelihood: “zero-truncated” binomial. Single parameter p.
- Full likelihood: binomial likelihood (has a term for n0 “all zero” encounter histories)

Up next: Bayesian analysis

- We analyze the full likelihood using a method known as data augmentation. This creates a “zero-inflated” binomial model.

BAYESIAN ANALYSIS OF CLOSED CAPTURE-RECAPTURE MODELS

- If N is known, Model M0 is just a logistic regression:

```
model {

p~dunif(0,1)

for (i in 1:N){
      y[i]~dbin(p,K)
    }


}
```

But N is not known. Conceptually we could just put a prior on N, e.g., N ~ Dunif(0, 1000), and analyze the model using standard methods of MCMC

However, the size of the data set, N, is a parameter of the model so as N is updated in the MCMC algorithm the size of the data set must change. Can't do this in WinBUGS/JAGS.

Bayesian analysis of closed population models

- Prior distributions:
  - N ~ Dunif(0, M), for M some big number
  - p ~ unif(0,1)

Not amenable to a naïve implementation by MCMC (esp in BUGs/JAGS) because N, a parameter, the number of individual effects, is unknown. "variable dimension parameter space"

- Therefore:
  - RJMCM/"Trans-dimensional" Gibbs sampling
  - Data augmentation <- easier, can be done in BUGS

DATA AUGMENTATION: HEURISTIC

- N ~ Dunif(0,M) implies a "data set" with M-n all-zero encounter histories. Some of the y=0 observations correspond to real individuals and some of them do not.
- Implementation: We add too many zeroes to the dataset – creating a zero-inflated version of the known-N dataset
- Model for the augmented data set is a zero-inflated binomial
- **THIS IS AN OCCUPANCY MODEL!**

HEURISTIC DEVELOPMENT

Occupancy data		Model M0		Model M0 + DA	
Site	|- occasion -|	Ind.	|- occasion -|	Ind.	|- occasion -|
1	0 1 0 1 1	1	0 1 0 1 1	1	0 1 0 1 1
2	0 0 1 0 0	2	0 0 1 0 0	2	0 0 1 0 0
3	1 1 0 0 0	3	1 1 0 0 0	3	1 1 0 0 0
4	0 0 1 1 0	4	0 0 1 1 0	4	0 0 1 1 0
5	0 1 1 1 1	5	0 1 1 1 1	5	0 1 1 1 1
6	0 0 1 1 0	6	0 0 1 1 0	6	0 0 1 1 0
7	1 1 1 1 1	7	1 1 1 1 1	7	1 1 1 1 1
8	1 0 1 1 0	8	1 0 1 1 0	8	1 0 1 1 0
	0 0 0 0 0				**0 0 0 0 0**
	0 0 0 0 0				**0 0 0 0 0**
	0 0 0 0 0				**0 0 0 0 0**
	0 0 0 0 0				**0 0 0 0 0**
	0 0 0 0 0				**0 0 0 0 0**
M	**0 0 0 0 0**			**M**	**0 0 0 0 0**

Zeros are observed. Allocate zeros to "fixed" and "sampling"

Zeros are NOT observed. How many "sampling" zeros are there?

Bound N <= M where M is fixed.

Treat Model M0 as an occupancy model.

DA AND OCCUPANCY MODELS

- DA makes capture-recapture models the same as occupancy models.
- The parameter ψ replaces population size N. They are related as follows: $N \sim \text{Binomial}(M, \psi)$

WHY CAN WE DO THIS?

- N ~ Unif(0,M)
- Same as:
  - N|psi ~ Bin(M, psi)   M = fixed
  - psi ~ uniform(0,1)

  This 2-part prior implies: N ~ Uniform(0,M) , standard distribution theory result
- Same as:
  - z[i] ~ Bern(psi) for i=1,2,…,M "data augmentation variables"
  - y[i] ~ Bern(p*z[i])
  - psi ~ dunif(0,1)   "data augmentation parameter"
- **The augmented data create a super-population of individuals available to be "recruited" by the MCMC algorithm.**

Fort Drum bear data

- Fit Model M0 in WinBUGS and JAGS using data augmentation

SUMMARY OF PART 1

- The essence of closed CR models is the binomial observation model for encounter frequencies
- Data augmentation is something you are probably unfamiliar with but it is really easy to analyze CR models using MCMC (esp. in BUGS).
- DA converts all capture-recapture models to "zero-inflated" models of one sort or another.
- We analyze all CR and SCR models using data augmentation. [even when we write our own code!]